

Effective theories and black hole production in warped compactifications

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Abstract

We investigate aspects of the four-dimensional effective description of brane world scenarios based on warped compactification on anti-de Sitter space. The low-energy dynamics is described by visible matter gravitationally coupled to a “dark” conformal field theory. We give the linearized description of the 4d stress tensor corresponding to an arbitrary 5d matter distribution. In particular a 5d falling particle corresponds to a 4d expanding shell, giving a 4d interpretation of a trajectory that misses a black hole only by moving in the fifth dimension. Breakdown of the effective description occurs when either five-dimensional physics or strong gravity becomes important. In scenarios with a TeV brane, the latter can happen through production of black holes near the TeV scale. This could provide an interesting experimental window on quantum black hole dynamics.

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1. Introduction

It is an old idea that, as an alternative to compactification, the observed Universe instead lives on a brane in a higher-dimensional space. Such “branification” scenarios had however until recently been hard to realize, largely because of the difficulty of recovering four-dimensional gravitational dynamics. Two new approaches have changed this and at the same time suggested new views of the origin of the hierarchy of scales in physics. The first, pursued by [1], is a hybrid of branification and compactification, in which matter is confined to a brane and then large-radius compactification of the extra dimensions yields four-dimensional gravity at long distances.

A more recent approach utilizes warped compactifications to achieve effectively four-dimensional gravitational dynamics. A outline of such a picture has been provided by the RSII model[2]. This utilizes a “Planck brane” that serves as the boundary of five-dimensional anti-de Sitter space, and the curvature of anti-de Sitter space effectively “localizes” low-energy gravity to the brane. Related models are the RSI model[3] in which AdS is terminated above the horizon by a “negative tension brane,” and the model of Lykken and Randall [4] in which visible sector matter lives on a probe brane. None of these are fundamental pictures as they do not provide a microscopic dynamics for the Planck, “negative-tension,” and probe branes, but recent work in string theory has begun to provide descriptions of such objects. In particular [5] has given a geometrical realization of an object akin to a Planck brane, and [6,7] have provided geometrical realizations of objects similar to “negative-tension” branes. At the same time, these models have been connected to renormalization group flows in four-dimensional gauge theories through the AdS/CFT correspondence.

In providing a new view of the hierarchy problem, either through large radius or other geometrical mechanisms, these scenarios suggest the exciting possibility that quantum gravity effects could be observed at scales far below the usual Planck scale, and perhaps even near the TeV scale. They also suggest the possibility of interesting new gravitational phenomena, particularly in scenarios with infinite extra dimensions (*e.g.* RSII) and with non-trivial curvature and horizon structure of the resulting spacetime.

Some aspects of this gravitational dynamics has been studied in [8-11]. In particular, [11] studied linearized gravity in the RSII scenario, and gave both prescriptions for computing propagators and a general picture of the structure of black holes bound to the Planck brane. The latter were found to be pancake-like objects, whose transverse sizes

are logarithmically smaller than their four-dimensional Schwarzschild radii. Cosmology of these scenarios has also been extensively studied (see *e.g.* [12-15]) with suggestions that they offer new approaches to the cosmological constant problem [16-25].

Many open questions remain, however, in the RSII scenario and its variants. One set of questions centers on the four-dimensional representation of the five-dimensional dynamics. In particular, localization of gravity is not complete and in the RSII scenario there is a gapless spectrum of analogs to Kaluza-Klein modes that are weakly coupled to excitations on the brane. Therefore a four-dimensional low-energy effective field theory does not follow from the usual Kaluza-Klein reasoning, and so one challenge has been to deduce what this effective theory is. It has previously been argued[5,26,27,11,28] that the bulk dynamics can be replaced via the AdS/CFT correspondence by a conformal field theory on the brane, and this suggests an answer, namely that the effective field theory is provided by conformal field theory coupled to the visible sector solely through gravity. This paper amplifies on this statement, clarifies the role of the cutoff, which in RSII is expected to be at the AdS radius scale, and provides one entry in the map between the five- and four-dimensional descriptions by computing a linearized approximation to the four-dimensional stress tensor corresponding to an arbitrary five-dimensional matter distribution. This stress tensor is both conserved and traceless. Corresponding statements should hold for other warped compactification scenarios, using realizations of the AdS/CFT correspondence in more general warped compactifications.

Given the novelties of the gravitational dynamics, for example the above picture of black holes, one is also prodded to investigate whether this field theory has unusual properties. For example, consider the following question[11]:¹ suppose that a particle is launched towards a black hole on the brane with zero four-dimensional impact parameter, but such that it follows a trajectory that misses the black hole through the fifth-dimension. Does this correspond in the four-dimensional perspective to matter that enters a black hole and exits the opposite side? This would surely be a radical departure from usual four-dimensional effective theory!

However, standard AdS/CFT reasoning suggests a more mundane answer. In the UV/IR correspondence outlined in [29], a state deep in AdS corresponds to a state in the far infrared of the corresponding field theory. This suggests that a falling particle corresponds to a state that spreads. Indeed, using our results for the stress tensor we find

¹ This question was asked by L. Susskind.

that in the four dimensional description, the falling particle corresponds to an expanding shell of CFT matter. The condition that the five-dimensional trajectory misses the black hole becomes the four-dimensional statement that the shell misses by expanding to a size larger than the black hole.

It is important to emphasize that the CFT description is an effective description, and another interesting set of questions therefore regards breakdown of the effective field theory and the question of whether strong gravitational dynamics – for example black hole formation – is observable at scales far below the four-dimensional Planck scale. We investigate the scales at which scattering experiments would be expected to encounter dynamics beyond the four-dimensional description in the three scenarios outlined, RSII, the probe brane scenario, and terminated AdS. In particular, in the latter scenario with a certain set of assumptions it appears possible to create black holes that decay into observable matter in scattering experiments in the vicinity of the TeV scale. This exciting possibility deserves more theoretical investigation; in particular through construction of concrete models with the required properties.

In outline, section II of this paper discusses conformal field theory as the 4d low-energy effective theory of RSII. Section III computes the linearized effective stress tensor of bulk matter, as well as solving a corresponding simpler problem of the 4d scalar profile of a five-dimensional scalar source. It also elaborates on the black hole flyby scenario mentioned above. Section IV then discusses questions of the scale of breakdown for the 4d effective theories, and of the possibility of low-energy black hole production. Section V closes with conclusions.

We have been informed that related work in progress[30] also addresses issues of black hole production and corrections to the effective theory in TeV brane scenarios.

2. The effective theory of RSII

We begin with a quick review of the RSII scenario, and of its transcription into conformal field theory via the AdS/CFT correspondence[26,27,11] in which we will offer some refinements. The upshot of this discussion is that the low-energy effective field theory for the RSII scenario consists of visible 4d matter gravitationally coupled to dark matter described by a cutoff CFT. Subsequent sections will explore consequences and extensions of this picture.

The RSII scenario is of course just an example of a much broader class of warped compactifications, which have recently been widely studied both in the context of model building, and in the context of string theory and the correspondence between renormalization group flows and supergravity geometries. While many of our comments will be made within the framework of this greatly simplified example (for which the only known microscopic construction is [5]), corresponding arguments should apply to other models including those with stringy realizations. In particular later sections will also comment on other variants of the RSII scenario (those with a terminated AdS space or with a probe or “TeV” brane) and their possible stringy realizations.

We therefore begin by considering the geometry with a single “Planck” brane. Although our central interest is dimension $d = 4$, most of the relevant formulas easily generalize and will be given in arbitrary dimension. We assume that matter fields, denoted by ψ , live only on this Planck brane. The action is

$$S = \int d^{d+1}X \sqrt{-G} (M^{d-1} \mathcal{R} - \Lambda) + \int d^d x \sqrt{-\gamma} [\mathcal{L}(\gamma, \psi) - \tau] \quad (2.1)$$

where G , M , \mathcal{R} , and Λ are the $d + 1$ dimensional metric, Planck mass, curvature scalar, and cosmological constant respectively, γ is the induced metric on the Planck brane, \mathcal{L} is the action of matter on the brane, and τ is the brane tension. The bulk AdS metric is

$$ds^2 = \frac{R^2}{z^2} (dz^2 + dx_d^2) \quad (2.2)$$

in $d + 1$ -dimensional coordinates $X = (x, z)$; here dx_d^2 is the d -dimensional Minkowski metric and the AdS radius R is given by

$$R = \sqrt{\frac{-d(d-1)M^{d-1}}{\Lambda}}. \quad (2.3)$$

The brane tension is fine tuned to the value

$$\tau = \frac{4(d-1)M^{d-1}}{R} \quad (2.4)$$

in order to maintain a Poincaré invariant Planck brane. We may take the Planck brane to reside at an arbitrary elevation $z = \rho$.

As argued in [3,2,11], at long distances compared to R , the gravitational dynamics appears d -dimensional. However, there is also a gapless spectrum of weakly-coupled bulk modes. An obvious question is what serves as a d -dimensional low-energy effective field

theory describing the dynamics. Within string theory, an answer to this is provided by the conjectured AdS/CFT correspondence [26,27,11].

To see this, recall that the AdS/CFT correspondence equates the $d + 1$ -dimensional bulk gravity (or more precisely, string theory) functional integral to a generating function in the CFT. A regulator is provided by excluding the AdS volume outside $z = \rho$. Suppose that we put the fluctuating metric in a gauge such that near this boundary

$$ds^2 = \frac{R^2}{z^2} [dz^2 + g_{\mu\nu}(z, x) dx^\mu dx^\nu] . \quad (2.5)$$

The induced metric γ on the boundary $z = \rho$ is thus

$$ds_d^2 = \frac{R^2}{\rho^2} g_{\mu\nu}(\rho, x) dx^\mu dx^\nu \equiv \gamma_{\mu\nu} dx^\mu dx^\nu . \quad (2.6)$$

Define the functional integral over bulk metrics G for fixed boundary metric γ as

$$Z[\gamma, \rho] = \int_{\gamma} \mathcal{D}G e^{i \int d^{d+1}X \sqrt{-G} (M^{d-1} \mathcal{R} - \Lambda) + 2i M^{d-1} \int d^d x \sqrt{-\gamma} K} \quad (2.7)$$

where K is the extrinsic curvature of the boundary. The AdS/CFT correspondence then states that for small fluctuations about the flat boundary geometry, $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

$$\lim_{\rho \rightarrow 0} e^{-i S_{\text{grav}}[\gamma]} Z[\gamma, \rho] = \left\langle e^{i \int h_{\mu\nu} T^{\mu\nu}} \right\rangle_{\text{CFT}} . \quad (2.8)$$

Here S_{grav} is a counterterm action formed purely from the induced metric γ [31,32]; in the case $d = 4$

$$S_{\text{grav}} = \int d^4x \sqrt{-\gamma} \left[\frac{6M^3}{R} + \frac{RM^3}{2} \mathcal{R}(\gamma) - 2M^3 R^3 \log(\rho) \mathcal{R}_2(\gamma) \right] \quad (2.9)$$

where

$$\mathcal{R}_2 = -\frac{1}{8} \mathcal{R}_{\mu\nu} \mathcal{R}^{\mu\nu} + \frac{1}{24} \mathcal{R}^2 . \quad (2.10)$$

While the AdS/CFT correspondence was originally stated in terms of small fluctuations, a natural assumption is that it extends to more general boundary geometries. We therefore assume that the CFT generating functional can be written as a functional integral over the CFT degrees of freedom, which we collectively denote χ , in the background metric $g_{\mu\nu}$, and that the correspondence thus becomes

$$\lim_{\rho \rightarrow 0} e^{-i S_{\text{grav}}[\gamma]} Z[\gamma, \rho] = \int \mathcal{D}\chi e^{i \int d^d x \sqrt{-g} \mathcal{L}_{\text{CFT}}(g_{\mu\nu}, \chi)} . \quad (2.11)$$

Following the ideas of the UV/IR correspondence[29], we connect this with the RS scenario by extending the conjecture to a statement with a finite cutoff, and assume that

$$e^{-iS_{\text{grav}}[\gamma]}Z[\gamma, \rho] = \int [\mathcal{D}\chi]_{\rho} e^{i \int d^d x \sqrt{-g} \mathcal{L}_{\text{CFT}}(g_{\mu\nu}, \chi)} \quad (2.12)$$

where the on the RHS ρ provides the cutoff scale for the CFT. While a precise description of this cutoff in the language of the CFT is not known, for sake of intuition one may imagine that it is for example given by only considering fluctuations on scales Δx such that

$$g_{\mu\nu} \Delta x^{\mu} \Delta x^{\nu} > \rho^2 . \quad (2.13)$$

In particular, notice that since the only dependence of the CFT on the scale of the metric is through the cutoff, this implies

$$\int [\mathcal{D}\chi]_{\rho} e^{i \int d^d x \sqrt{-g} \mathcal{L}_{\text{CFT}}(g_{\mu\nu}, \chi)} = \int [\mathcal{D}\chi]_R e^{i \int d^d x \sqrt{-\gamma} \mathcal{L}_{\text{CFT}}(\gamma_{\mu\nu}, \chi)} \quad (2.14)$$

where on the RHS the cutoff is thought of as restricting to fluctuations with

$$\gamma_{\mu\nu} \Delta x^{\mu} \Delta x^{\nu} > R^2 . \quad (2.15)$$

From (2.12) and (2.14) we therefore see that the integral over the bulk modes can be replaced by a correlator in the CFT, as originally proposed in [26,27,11], with a cutoff given by R . Specifically, d -dimensional dynamics is summarized by a functional integral of the form

$$\int [\mathcal{D}\gamma \mathcal{D}\psi \mathcal{D}\chi]_R e^{i \int d^d x \sqrt{-\gamma} [\frac{1}{2} \mathcal{L}(\gamma, \psi) + \mathcal{L}_{\text{CFT}}(\gamma, \chi) + \mathcal{L}_{\text{grav}}(\gamma) - \tau]} (\dots) . \quad (2.16)$$

For consistency with the cutoff (2.15) the other modes also presumably should have a corresponding cutoff, as indicated. One consistency check on this approach is cancellation of the brane tension τ by the corresponding term in S_{grav} , using (2.4). This indicates that the low-energy effective field theory for the system, up to the scale determined by R , is the theory of brane-matter gravitationally coupled to “dark” matter described by the CFT. The d -dimensional Planck mass follows from the d -dimensional version of (2.9), and is given by

$$M_d^{d-2} = \frac{RM^{d-1}}{d-2} . \quad (2.17)$$

3. Effective stress tensor of bulk matter

We now investigate some of the consequences of the above identification of the CFT as the low-energy effective field theory for the RSII scenario. In particular, we start by giving an entry in the bulk to boundary dictionary, by computing a linearized approximation to the CFT stress tensor corresponding to a perturbation in the bulk. We then investigate the particular case of a particle freely falling into the bulk.

Using this calculation, we discuss a test of the AdS/CFT correspondence and of our effective description of RSII: suppose that we shoot a particle towards a black hole with zero 4d impact parameter, but such that it will miss the black hole through the z direction. How does a 4d observer understand the failure of the black hole to absorb the particle?

3.1. General results

In this subsection we turn to the problem of deriving the d -dimensional brane stress tensor that corresponds to a general $d+1$ -dimensional bulk matter distribution. In general this is a difficult problem, requiring solution of the bulk Einstein equations, so we will only give a linear treatment.

The basic strategy is as follows. Ref. [11] computes the linearized bulk gravitational field of a general matter perturbation. This in particular gives the linearized metric and therefore Einstein tensor induced on the brane. We can then read off the matter stress tensor from the right hand side of the d -dimensional Einstein equations along the brane.

Although the resulting stress tensor has a number of special properties, we have not yet found a particularly illuminating expression for it. However, in the next subsection we specialize to the case of a particle falling into the bulk; in the long distance approximation the corresponding stress tensor simplifies substantially.

In studying gravitational perturbations it proves convenient to introduce the proper “height” coordinate y , given by

$$z = Re^{y/R} \tag{3.1}$$

in terms of which the linearized metric takes the form

$$ds^2 = dy^2 + e^{-2y/R}(\eta_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu . \tag{3.2}$$

Eqs. (3.20), (3.24), and (3.26) of [11] then give the linearized bulk Einstein equations in terms of the metric perturbation $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}h\eta_{\mu\nu}$ as:

$$\partial_y \left(e^{-2y/R} \partial_y h \right) = \frac{1}{(d-1)M^{d-1}} \left[T_\mu^\mu - (d-2)e^{-2y/R} T_y^y \right] , \tag{3.3}$$

$$\partial_y \partial^\nu h_{\mu\nu} = \partial_y \partial_\mu h + \frac{T_\mu^y}{M^{d-1}} , \quad (3.4)$$

and

$$\begin{aligned} \square \bar{h}_{\mu\nu} &= \frac{\eta_{\mu\nu}}{2} e^{yd/R} \partial_y (e^{-yd/R} \partial_y h) \\ &+ e^{2y/R} (-\eta_{\mu\nu} \partial^\lambda \partial^\sigma \bar{h}_{\lambda\sigma} + \partial^\lambda \partial_\mu \bar{h}_{\nu\lambda} + \partial^\lambda \partial_\nu \bar{h}_{\mu\lambda}) \\ &- \frac{e^{2y/R}}{M^{d-1}} T_{\mu\nu} . \end{aligned} \quad (3.5)$$

The right hand side of (3.5) is determined by the stress tensor and the solutions of eqs. (3.3), (3.4). This equation can then be solved for $h_{\mu\nu}$ using the scalar Neumann Green function Δ_{d+1} , satisfying

$$\begin{aligned} \square \Delta_{d+1}(X, X') &= \frac{\delta^{d+1}(X - X')}{\sqrt{-G}} , \\ \partial_y \Delta_{d+1}(X, X')|_{y=0} &= 0 , \end{aligned} \quad (3.6)$$

and which was derived in [11]. In the present situation we need the retarded propagator rather than the Feynman propagator; the relation between these and approximate expressions for them are given in the appendix. The resulting expression for the metric has three terms arising from the three lines of (3.5). However, the second term is inessential as a short calculation shows it to be pure gauge on the brane. Therefore its contribution drops when we compute the Einstein tensor on the brane.

One must also specify boundary conditions at the brane; in the case of a surface stress tensor

$$T_{\mu\nu}^{\text{brane}} = S_{\mu\nu}(x) \delta(y) , \quad T_{yy}^{\text{brane}} = T_{y\mu}^{\text{brane}} = 0 \quad (3.7)$$

these become

$$\partial_y (h_{\mu\nu} - \eta_{\mu\nu} h)|_{y=0} = -\frac{S_{\mu\nu}(x)}{2M^{d-1}} . \quad (3.8)$$

In order to simplify the resulting expression for the metric, it is useful to rewrite the scalar Green function in terms of a new function F as

$$\Delta_{d+1}(y, x; y', x') = e^{(d-2)y'/R} \partial_{y'} \left[e^{-(d-2)y'/R} F_{y'}(y; x - x') \right] . \quad (3.9)$$

One nice property of this redefinition is immediate: one readily checks that

$$\int_0^\infty dy' e^{(2-d)y'/R} \Delta_{d+1}(X, X') = -F_0(y; x - x') \quad (3.10)$$

satisfies the equation for the d -dimensional Green function, and so

$$F_0(y; x - x') = -\Delta_d(x, x') . \quad (3.11)$$

Using this and integrating by parts gives the contribution to $\bar{h}_{\mu\nu}$ from the first line in (3.5) as

$$\bar{h}_{\mu\nu}^{(1)}(x, y) = \frac{\eta_{\mu\nu}}{2(d-1)M^{d-1}} \int dV' \partial_{y'} F_{y'}(y; x - x') \left[e^{2y/R} T_\mu^\mu(X') - (d-2) T_y^y(X') \right] . \quad (3.12)$$

Eq. (3.12) combines with the terms induced by the stress tensor and the surface stress (3.7) to give a complete expression of the form

$$\begin{aligned} \bar{h}_{\mu\nu} = & -\frac{1}{M^{d-1}} \int dV' e^{dy'/R} \partial_{y'} \left[e^{-(d-2)y'/R} F_{y'}(y; x - x') \right] T_{\mu\nu}(X') \\ & + \frac{\eta_{\mu\nu}}{2(d-1)M^{d-1}} \int dV' \partial_{y'} F_{y'}(y; x - x') \left[e^{2y/R} T_\mu^\mu(X') - (d-2) T_y^y(X') \right] \\ & + \bar{h}_{\mu\nu}^S + \bar{h}_{\mu\nu}^{\text{gauge}} . \end{aligned} \quad (3.13)$$

Here $\bar{h}_{\mu\nu}^{\text{gauge}}$ is the piece that is pure gauge on the brane, mentioned above, and $\bar{h}_{\mu\nu}^S$ is the contribution due to the surface stress (we will see an example of this shortly).

For simplicity consider a purely bulk distribution ($S_{\mu\nu} = 0$). The four-dimensional effective stress tensor is readily computed from (3.13) via Einstein's equations,

$$T_{\mu\nu}^{\text{eff}} = 2M_d^{d-2(d)} \mathcal{G}_\nu^\mu = -\frac{RM^{d-1}}{d-2} (\partial^2 \bar{h}_{\mu\nu} + \eta_{\mu\nu} \partial^\alpha \partial^\beta \bar{h}_{\alpha\beta} - \partial_\mu \partial^\alpha \bar{h}_{\alpha\nu} - \partial_\nu \partial^\alpha \bar{h}_{\alpha\mu})|_{y=0} , \quad (3.14)$$

with $\bar{h}_{\mu\nu}$ given by (3.13). The contribution of $\bar{h}_{\mu\nu}^{\text{gauge}}$ drops out.

One may expand out the expression (3.14) to write it explicitly in terms of the bulk stress tensor $T_{\mu\nu}$:

$$\begin{aligned} T_{\mu\nu}^{\text{eff}} = & \frac{R}{d-2} \int dV' \left\{ e^{dy'/R} \partial_{y'} \left[e^{-(d-2)y'/R} F_{y'}(0; x - x') \right] (\partial^2 T_{\mu\nu} + \eta_{\mu\nu} \partial^\alpha \partial^\beta T_{\alpha\beta} \right. \\ & \left. - \partial^\alpha \partial_\mu T_{\alpha\nu} - \partial^\alpha \partial_\nu T_{\alpha\mu}) \right. \\ & \left. + \frac{1}{d-1} \partial_{y'} F_{y'}(0; x - x') (\partial_\mu \partial_\nu - \eta_{\mu\nu} \partial^2) \left[e^{2y'/R} T_\mu^\mu - (d-2) T_y^y \right] \right\} . \end{aligned} \quad (3.15)$$

Note that $T_{\mu\nu}^{\text{eff}}$ satisfies two important properties. First, from its construction and the Bianchi identities, it is conserved:

$$\partial^\mu T_{\mu\nu}^{\text{eff}} = 0 . \quad (3.16)$$

Secondly, one may readily verify that it is traceless,

$$\eta^{\mu\nu} T_{\mu\nu}^{\text{eff}} = 0 , \quad (3.17)$$

which accords nicely with its interpretation as arising from a conformal field theory on the brane. Indeed, this easily follows from the (yy) Einstein equation, which states (*cf.* [11] eq. (3.14))

$${}^{(d)}\mathcal{R} + \frac{(d-1)}{R} \partial_y h \epsilon(y) = -\frac{T_y^y}{M^{d-1}} \quad (3.18)$$

where $\epsilon(y)$ is a step function. On the brane $\partial_y h$ and T_y^y vanish (the former by (3.8)), implying ${}^{(d)}\mathcal{R}(y=0) = 0$, and thus $T^{\text{eff}} = 0$.

Note that in the above discussion we have said nothing about bending of the brane, which was described in [10,11]. The reason for this is that we are interested in the metric on the brane, and for this it is best to work in a gauge where the brane is straight. In [11] the resulting metric was computed by first working in the bent gauge, and then transforming back, but an equivalent result is found by working directly in the straight gauge.²

3.2. Matter on the brane

In order to illustrate this equivalence – and because the result will be used in the next subsection – we'll compute the linearized metric and effective stress tensor due to matter on the brane in this approach. Specifically, suppose that there is a surface stress of the form (3.7), but that otherwise $T_{IJ} = 0$. The field eqs. (3.3)-(3.5) should then be solved subject to the boundary conditions (3.8). By tracing the latter can be rewritten in terms of \bar{h} , and take the form

$$\partial_y \bar{h}_{\mu\nu}|_{y=0} = -\frac{1}{2M^{d-1}} \left[S_{\mu\nu} - \frac{\eta_{\mu\nu}}{2(d-1)} S \right] . \quad (3.19)$$

By Green's theorem these give a contribution

$$\bar{h}_{\mu\nu}^S(X) = -\frac{1}{2M^{d-1}} \int d^d x' \Delta_{d+1}(X; 0, x') \left[S_{\mu\nu}(x') - \frac{\eta_{\mu\nu}}{2(d-1)} S(x') \right] \quad (3.20)$$

² For purposes of measurements on the brane, the apparent breakdown of the linearized approximation at $y \rightarrow \infty$ may be ignored; for another treatment of these matters see [33].

to the metric. As above, the second term on the RHS of (3.5) is pure gauge, and the third term vanishes, so the remaining contribution comes from the first term. The trace equation (3.3) and the boundary condition (3.8) imply

$$\partial_y h = \frac{e^{2y/R}}{2(d-1)M^{d-1}} S , \quad (3.21)$$

which gives a contribution

$$\bar{h}_{\mu\nu}^{(1)} = \frac{\eta_{\mu\nu}(2-d)}{4(d-1)RM^{d-1}} \int d^d x' S(x') \int dy' e^{(2-d)y/R} \Delta_{d+1}(X, X') . \quad (3.22)$$

The integral over y' is eliminated by using the identities (3.10) and (3.11), and the combined expressions (3.20) and (3.22) yield

$$\begin{aligned} \bar{h}_{\mu\nu}(x) = & -\frac{1}{2M^{d-1}} \int d^d x' \left\{ \Delta_{d+1}(x, 0; x', 0) S_{\mu\nu}(x') - \right. \\ & \left. \eta_{\mu\nu} \left[\Delta_{d+1}(x, 0; x', 0) - \frac{(d-2)}{R} \Delta_d(x, x') \right] \frac{S_\lambda^\lambda(x')}{2(d-1)} \right\} \end{aligned} \quad (3.23)$$

in agreement with [11]. In particular, this expression may be evaluated for a stress tensor corresponding to a point mass at rest on the brane at $\vec{x} = 0$,³

$$T_{tt} = 2m\delta^{d-1}(x)\delta(y) . \quad (3.24)$$

Using the long-distance expansion of the propagator[11],

$$\Delta_{d+1}(x, 0; x', 0) = \frac{d-2}{R} \Delta_d(x, x') \left[1 + \left(\frac{R^{d-2}}{r^{d-2}} \right) \right] , \quad (3.25)$$

this gives the $d = 4$ expression

$$\bar{h}_{tt} = \frac{m}{2\pi RM^3 r} \left[1 + \mathcal{O}\left(\frac{R^2}{r^2}\right) \right] , \quad \bar{h}_{ij} = \mathcal{O}\left(\frac{mR}{M^3 r^3}\right) . \quad (3.26)$$

3.3. The falling particle

The above expression (3.15) for the stress tensor appears rather complicated, but simplifies significantly in the long-distance limit. To illustrate this, we compute the effective

³ The extra factor of two is present because of the orbifold boundary conditions, and compensates the 1/2 in (2.16).

stress tensor of a particle falling into the bulk. (The corresponding approximate metric has also been computed by Gregory, Rubakov, and Sibiryakov in [34].) This case will also apply to our later discussion of black hole flybys; by performing a boost along the brane we get a trajectory that can sail behind a black hole through the extra dimension.

Concretely, consider a particle of mass m that is stuck to the brane at $\vec{x} = 0$ until time $t = 0$ and then released and allowed to freely fall into the bulk. The trajectory for $t > 0$ is easily seen to be given by the equation

$$z^2 - t^2 = R^2 . \quad (3.27)$$

For $t < 0$ the only nonzero component of the stress tensor is given by (3.24). For $t > 0$ the stress tensor is given by the general formula

$$T_{IJ} = m \frac{dX_I}{d\tau} \frac{dX_J}{dt} \frac{\delta^{d-1}(x - x(t))\delta(y - y(t))}{\sqrt{-G}} , \quad (3.28)$$

which in the present case gives nonvanishing components

$$T_{tt} = m e^{(d-2)y/R} \delta^{d-1}(\vec{x}) \delta(y - y(t)) , \quad (3.29)$$

$$T_{yy} = m \frac{t^2}{R^2} e^{(d-2)y/R} \delta^{d-1}(\vec{x}) \delta(y - y(t)) , \quad (3.30)$$

and

$$T_{ty} = -m \frac{t}{R} e^{(d-2)y/R} \delta^{d-1}(\vec{x}) \delta(y - y(t)) . \quad (3.31)$$

Therefore the contribution to the metric from the trajectory for $t < 0$ is a special case of the general surface-stress result of the preceding subsection, (3.23), with

$$S_{tt} = 2m \delta^{d-1}(x) \theta(-t) , \quad S_{\mu i} = 0 . \quad (3.32)$$

The contribution to the metric from the second half of the trajectory is given by our formula (3.13). Specifically, rewriting the $t > 0$ stress tensor as

$$T_{IJ} = \mathcal{S}_{IJ}(t, y) \delta(y - y(t)) , \quad (3.33)$$

we find

$$\begin{aligned} \bar{h}_{\mu\nu}^>(x) = & -\frac{1}{M^{d-1}} \int_{t'>0} d^d x' \left\{ \partial_y \left[e^{-(d-2)y/R} F_y(0; x - x') \right] \mathcal{S}_{\mu\nu}(x', y) \right. \\ & - \frac{\eta_{\mu\nu}}{2(d-1)} e^{-(d-2)y/R} \partial_y F \left[\mathcal{S}_{\mu}^{\mu} - (d-2)e^{-2y/R} \mathcal{S}_y^y \right] \Big|_{y=y(t)} \\ & \left. + \bar{h}_{\mu\nu}^{\text{gauge}} \right\} . \end{aligned} \quad (3.34)$$

The expression for the effective stress tensor follows directly from computing the Einstein tensor (3.14) from these expressions for the linearized metric. In order to gain some intuition for this expression, consider the approximation of distances and times much greater than the AdS scale R , which we've seen is the cutoff for the effective theory:

$$x^2 - t^2 \gg R^2 . \quad (3.35)$$

In this limit the Green function simplifies dramatically (see appendix),

$$F_y(0; x - x') \simeq \frac{1}{2\pi} \delta(z^2 + (x - x')^2) \theta(t - t') , \quad (3.36)$$

and the trajectory (3.27) becomes

$$z = R e^{y/R} \simeq t . \quad (3.37)$$

Defining $r = |x|$, in $d = 4$ the approximate metric is then

$$\bar{h}_{\mu\nu} \simeq \frac{m}{2\pi M^3 R r} \delta_\mu^t \delta_\nu^t \quad (3.38)$$

for $r > t$, and

$$\bar{h}_{\mu\nu} \simeq \frac{m}{2\pi M^3 R} \left[\left(\frac{3}{2t} - \frac{\vec{x}^2}{2t^3} \right) \delta_\mu^t \delta_\nu^t + \frac{t^2 - \vec{x}^2}{4t^3} \eta_{\mu\nu} \right] \quad (3.39)$$

for $r < t$, as in [34]. A straightforward computation shows that the Einstein tensor of both of these metrics vanishes! Thus the effective stress tensor is concentrated on the surface where they match, $r = t$. This stress tensor is

$$T_{\mu\nu}^{\text{eff}} \simeq \frac{m}{4\pi t^2} \delta(t - r) u^\mu u^\nu \quad (3.40)$$

where $u^\mu = x^\mu / t$.

The effective stress tensor of the conformal field theory configuration describing a falling particle is thus concentrated on a thin shell of radius r which expands outward with time, $r = t$. We can estimate the thickness of the shell by investigating the size of the leading corrections in the limit (3.35). One readily sees that the metric is corrected at order R^2/t^2 , R^2/r^2 , both due to corrections to the trajectory and to the Green function. This suggests that the thickness of the shell of CFT matter is the expected $\mathcal{O}(R)$, the cutoff length scale.

This spreading behavior appears to be quite generic, as one might expect from the IR/UV duality of the AdS/CFT correspondence. Another example of this behavior is provided by a falling charged particle coupled to a bulk gauge field, as investigated in [35]. Indeed, an even simpler example is provided by a falling particle coupled to a bulk scalar field. Specifically, consider a Lagrangian

$$S = - \int dV \frac{1}{2} (\nabla \phi)^2 - q \int d\tau \phi(X(\tau)) \quad (3.41)$$

with a coupling of a bulk scalar field ϕ to a particle of scalar charge q falling along a trajectory $X(\tau)$. This determines the scalar field,

$$\phi(X) = q \int d\tau \Delta_{d+1}(X, X(\tau)) . \quad (3.42)$$

If we assume that the particle again follows the trajectory (3.27) and work at large distances as compared to R and with $d = 4$, then the field at $y = 0$ takes the approximate form

$$\phi(x, t) \simeq -\frac{q}{2\pi R} \left[\frac{1}{r} \theta(r - t) + \frac{4R^2 t}{(r^2 - t^2)^2} \theta(t - r) \right] \left[1 + \mathcal{O}\left(\frac{R}{r}\right)^2 \right] . \quad (3.43)$$

If we compute the effective source, $J = \square_4 \phi$, we find it vanishes except at $r = t$. Again, subleading $\mathcal{O}(R)$ corrections appear to smooth this into a shell of thickness R .

Note that similar behavior was found by Horowitz and Itzhaki [36], who investigated the CFT stress tensor corresponding to a particle moving geodesically in the full, infinite AdS. This work also found a shell expanding outward at the speed of light. Indeed, the two calculations are directly related in the infrared limit, as discussed in appendix B.

This behavior can also be understood directly in terms of the CFT using an argument due to Coleman and Smarr [37], which shows that a stress tensor that is conserved, traceless, and has positive energy density will be localized on the light cone. The basic idea for the proof is to show that the average squared energy radius,

$$\bar{r}(t)^2 = \frac{\int d^3x r^2 T_{00}}{\int d^3x T_{00}} \quad (3.44)$$

satisfies

$$\frac{d^2 \bar{r}^2}{dt^2} = 2 \quad (3.45)$$

from which it immediately follows that a configuration initially localized at a point will expand on the light cone. Ref. [36] argues that the argument extends even to the quantum case, where the energy density may be negative, as long as the total energy is positive.

These results neglect the backreaction of gravity on the outgoing shell. It would be interesting to understand what dynamics results when strong self gravitation of the shell is included.

Vanishing of the Einstein tensor for the metric (3.39) at first sight leads to another puzzle. Specifically, suppose we consider a “bounce” trajectory, where the particle follows the trajectory (3.27) for *all* time. The calculation of the metric above is modified by extending to the trajectory for $t < 0$, but still yields a stress tensor that vanishes everywhere. This contradicts our expectation of a shell that collapses and then reexpands. However, note that this computation is not complete. The z coordinates only cover the region outside the AdS horizon, and thus this calculation would miss the contribution of the piece of the trajectory behind the past horizon. If this is not included, energy-momentum conservation is violated at the horizon, and consequently gravity cannot be consistently coupled. A correct calculation includes this piece, but also requires more information about the structure of the Green function. Specifically, one needs to know what boundary conditions it obeys in the far past. In order to make predictions in the RSII scenario, one needs to understand the physics determining the boundary conditions at the past horizon. Correspondingly, in CFT language one needs to know in what state the CFT sector began.

3.4. Black hole flybys

We now have the necessary tools to discuss particles flying past black holes through the bulk; for simplicity we discuss the four-dimensional case. Specifically, suppose that there is a black hole of mass m located at $\vec{x} = 0$ and that a particle is shot at it with zero four-dimensional impact parameter, but is allowed to fall in z long enough that it misses the black hole by passing it in the z direction (see fig. 1). How does a four dimensional observer describe such an experiment, and in particular does one see radical departures from usual gravitational dynamics, such as matter entering and then escaping a 4d event horizon?

The answer to the latter question is, of course, no. Indeed, a black hole with mass and 4d Schwarzschild radius m has a horizon extending to $z_h \sim m$ in the bulk picture. In order for the particle to miss the black hole, the particle must have $z \gg m$ when $\vec{x} = 0$. As we’ve seen above, in the CFT description the particle corresponds to a shell of CFT matter. If it has reached $z \gg m$ by the time it reaches the black hole at $\vec{x} = 0$, then the shell has expanded to size $r \gg m$ by the time it has reached the black hole, and is continuing to expand outward. No novel physics need be invoked to explain why

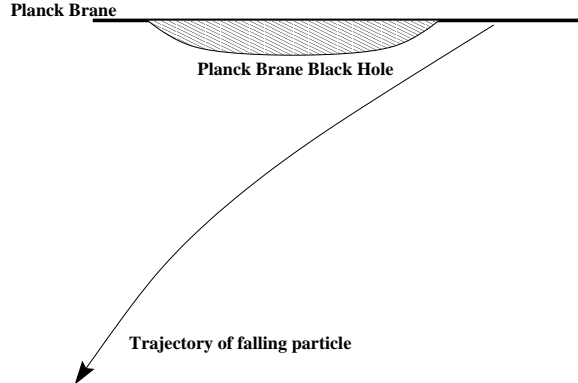


Fig. 1: A particle trajectory that misses a black hole on the brane because of its motion in the extra dimension.

the shell is not absorbed by the black hole.⁴ The process has a perfectly adequate four-dimensional effective description in terms of the matter conformal field theory coupled to four-dimensional gravity.

4. Breakdown of EFT; cutoffs, strong gravity, and black hole production

Section two argued that at low energies the RS scenario is equivalent to coupling ordinary matter to a hidden CFT. Section three provided illustrations of this statement. An obvious question regards the limitations of this description. At what scale does it fail? Is there any practical advantage or consequence of the five-dimensional description? And what conclusions can one draw about strong gravitational phenomena, such as production of black holes in high-energy scattering?

In this section we will first consider the scenario with a single Planck brane, and then comment on extensions of the discussion to scenarios with an added probe or “TeV” brane or with AdS terminated by a brane-like object at large z (like the “negative tension” brane proposal of [3]).

4.1. Scattering on the Planck brane

The four-dimensional effective action for the scenario with a single Planck brane is given in (2.16). Recall that the fundamental parameters determine the 4d Planck mass by

⁴ Note, however, that a small piece of the shell may be absorbed by the black hole; a quantum treatment of the bulk should yield a corresponding result.

the relation (2.17). We would like to understand what this scenario predicts for high-energy scattering.

The simplest assumption (if one is not trying to solve the hierarchy problem) is that the five-dimensional Planck scale and inverse AdS radius, and hence the four-dimensional Planck scale, are all comparable:

$$M \sim 1/R \sim M_4 . \quad (4.1)$$

In that case all new physics is clearly at the Planck scale. How much can this statement be relaxed? One would expect observable deviations in microgravity experiments – as in the scenario of [1] – for $R \sim 1mm$. This puts a lower bound of $M \gtrsim 10^8 GeV$ on the five-dimensional Planck scale.

Consider now high energy scattering of particles confined to the brane. From the bulk perspective, we see that at distances $\ll R$ the dynamics is effectively five-dimensional. This is mirrored in the four-dimensional description of (2.16); energies above $1/R$ are past the cutoff and the cutoff CFT description is incomplete.

Does this mean that we can see what a 4d observer would interpret as strong gravitational phenomena at energies just above $1/R$? Clearly not, except when the parameters satisfy $1/R \sim M$, in which case we are at Planckian 4d energies anyway. Consider for example black hole production. There are two types of black holes that one might produce. The first type is the AdS/Schwarzschild black hole, which moves freely in the bulk, and in general will fall towards the AdS horizon once produced. The threshold for producing such black holes is the 5d Planck energy $M \gtrsim 10^8 GeV$. The second type of black hole is bound to the brane, as described in [38,11]. The horizon radius of such a black hole is $r_h \sim m/M_4^2$; this should be larger than the 5d Planck length which implies $m > RM^2$. Since $RM \gtrsim 1$, the threshold is again at M or above.

From this discussion we see that scattering pushes beyond the cutoff scale at the threshold $1/R$ and in the bulk perspective begins to explore the extra dimension. While this may have visible consequences through production of the Kaluza-Klein modes, strong gravitational dynamics such as black hole production has a much higher threshold of M , which in the most “optimistic” scenario of $M \sim 10^8 GeV$ is still a long ways off.

4.2. Scattering on a probe brane

The preceding Planck-brane scenario is not favored from the viewpoint of low-energy phenomenology in any case, given the expected relation (4.1) between scales. Scenarios which try to generate the hierarchy via the exponential warp factor show more promise. Consider first the probe brane scenario of [4]. Here 4d matter is taken to reside on a “TeV” brane at an elevation $z = \rho_T$; the Planck brane is again at $z = \rho$. This brane must be stabilized by a mechanism such as in [39,40]. The 4d Planck mass is again given by (2.17), but matter on the TeV brane has its energy redshifted by ρ/ρ_T relative to the Planck brane. If $\rho/\rho_T \sim \text{TeV}/M_4$, this gives a mechanism to generate TeV scale effective masses from particles with fundamentally Planckian masses.

To elaborate on these comments, note that in giving a four-dimensional description of the physics it is necessary to specify a reference frame at a definite value of z in terms of which four-dimensional energies are measured. The natural frame to use is that of the Planck brane, as this is where the 4d graviton bound state is supported. Then if we consider an energy E_{prop} as measured by an observer at another value of z , it will be redshifted so that the energy in the frame of the Planck brane is $E = \rho E_{\text{prop}}/z$. In particular, a particle of mass m at rest in the frame at z will have an energy $m\rho/z$ relative to the Planck brane, and that will be interpreted as its four-dimensional mass.

The Lagrangian in this scenario is expected to take the form

$$S = \int d^{d+1}X \sqrt{-G} (M^{d-1} \mathcal{R} - \Lambda + \mathcal{L}_{\text{stab}}) + \int d^d x \left[\sqrt{-\gamma(x, \rho_T)} \mathcal{L}(\gamma(x, \rho_T), \psi) - \sqrt{-\gamma(x, \rho)} \tau \right] . \quad (4.2)$$

Here we denote by $\mathcal{L}_{\text{stab}}$ the Lagrangian of the stabilizing fields; we assume that beyond stabilizing the brane they don’t qualitatively affect our conclusions.

What is the CFT description of this scenario? Here we encounter subtleties beyond the derivation of (2.16). Specifically, the action depends on the metric at $z = \rho_T$. In attempting to relate the bulk functional integral to the boundary CFT we have to confront the non-trivial z -dependence of γ , and in particular give a CFT prescription for computing the metric in the bulk. We have not yet found a convincing prescription to derive such off-shell information from the AdS/CFT correspondence.

In the absence of such a prescription we will consider two approaches to this problem. The first is to work with long-wavelength excitations of the theory such that the simple scaling approximation

$$\gamma(x, \rho) \simeq \frac{\rho_T^2}{\rho^2} \gamma(x, \rho_T) \quad (4.3)$$

holds. We use this equation to replace $\gamma(x, \rho_T)$ by $\gamma(x, \rho)$ in the Lagrangian for matter on the TeV brane. This effectively rescales parameters of dimension δ in that Lagrangian by a factor $(\rho/\rho_T)^\delta$ (*c.f.* [4]). Rewriting the functional integral as in section two produces a 4d effective action analogous to (2.16) in the Planck brane scenario,

$$S_{TeV} = \int d^4x \sqrt{-\gamma} [\mathcal{L}(\gamma, \psi, m\rho/\rho_T) + \mathcal{L}_{\text{CFT}}(\gamma, \chi) + \mathcal{L}_{\text{grav}}(\gamma) - \tau] . \quad (4.4)$$

Here we have explicitly indicated the rescaling of a typical mass parameter in the matter Lagrangian. Again $\mathcal{L}_{\text{CFT}}(\chi)$ represents the Lagrangian of “dark” CFT matter, and $\mathcal{L}_{\text{grav}}$, given by (2.9), is the gravitational action.

The simple approximation (4.3) fails at short wavelengths, where the z dependence becomes non-trivial. This effect can be estimated from the long-distance expansion of the propagator[11],

$$\Delta(x, z; x', z') \sim \frac{1}{Rr^{d-2}} \left[1 + \frac{R^{d-2}}{r^{d-2}} + \frac{z^d}{r^d} + \frac{z^{2d}}{r^{2d}} \frac{r^{d-2}}{R^{d-2}} \right] \left[1 + \mathcal{O}\left(\frac{z^2}{r^2}, \frac{R^2}{r^2}\right) \right] . \quad (4.5)$$

In $d = 4$, the correction due to the last term becomes large at distances

$$r \sim \left(\frac{\rho_T}{\rho} \right)^{4/3} R , \quad (4.6)$$

or at about 10 Fermi for $\rho/\rho_T \sim TeV/M$.

In order to understand the origin of corrections at this scale, first let’s recall a similar phenomenon in the large scale compactification scenario of [1]. If one for example considers such a compactification with two extra dimensions of size $\mathcal{O}(mm)$, gravitational experiments performed at shorter scales reveal the six-dimensional nature of spacetime: the part of the four-dimensional effective Lagrangian describing the gravitational sector breaks down. One way of understanding this is to note that sources with shorter wavelengths than a millimeter will generically have coupling to the Kaluza-Klein modes that is comparable to the coupling to the gravitational zero mode; summing over these modes produces the six-dimensional gravitational field. While the gravitational part of the 4d effective action

breaks down, nonetheless the gauge part of the effective action remains four-dimensional up to scales of order a TeV where gravity itself becomes strongly coupled.

A similar phenomena occurs here. At scales given by (4.6), the couplings of the TeV brane matter to the continuum analogs of the Kaluza-Klein states become comparable to the coupling to the four-dimensional graviton. This means that in the gravitational sector the 4d effective theory fails, but of course the gauge part of the theory remains four-dimensional up to the TeV scale. The stress tensor of the TeV brane matter acts as the source for these couplings to the Kaluza-Klein modes.

Corresponding statements can be made in the CFT, and will tell us the form of the corrections to the action (4.4) that are responsible for its failure as a 4d effective description. In particular, we expect that corresponding to the couplings to the KK modes, a term is induced in (4.4) in which there is a direct coupling of the stress tensor of the TeV brane matter to the stress tensor of the CFT, and by scaling the coefficient of this should include a factor of $(\rho_T/\rho)^4$. Such terms are responsible for the breakdown of the gravitational part of the 4d effective theory.

A second approach would be to use the holographic renormalization group [41] to evolve the Lagrangian from the Planck brane to the TeV brane. We would expect this to produce similar results, namely a gravitationally coupled CFT with a cutoff scale ~ 10 MeV. We expect the $T_\chi T_\psi$ terms described above to be present in the Lagrangian at the Planck scale, and then to be rescaled by the renormalization group flow. A better understanding along these lines of the relationship between operators at different z would also clearly be illuminating for our fundamental understanding of holography in the AdS/CFT correspondence.

As in the scenario of [1], there is a distinction between the scale at which the 5d nature of gravity becomes important and the scale at which gravity becomes strongly coupled. A particularly interesting question is when do we expect to be able to manufacture configurations which would manifest signatures for black holes that we as four-dimensional observers could see?

Within the context of the TeV-brane scenario, there are again two kinds of black hole solutions known. The first is the AdS-Schwarzschild black hole. The minimum energy to create these should be $\mathcal{O}(M)$. A collision of TeV brane matter with a proper energy of this magnitude is a collision at the TeV energy scale as measured with respect to the four-dimensional observer. However, it appears that such black holes are not bound to the brane. In the probe-brane limit, where the gravitational backreaction is neglected,

this is manifest, but even taking into account the small energy density of the probe brane it seems likely that the binding of the black hole to the brane will be overcome by the gravitational pull of the black hole towards the AdS horizon.⁵ While a complete analysis of this requires detailed investigation of stabilized probe brane scenarios, it appears that such a black hole will therefore generically fall towards the AdS horizon, and that the 4d observer will therefore not perceive it as a black hole. In the CFT picture, such black holes will be perceived as complex excitations of the CFT which spread out over time, and it is very unlikely that their signature can be experimentally disentangled from other excitations of the CFT.

The second type of solution is the black hole on the Planck brane. These are truly perceived as 4d black holes. However, given the relation (4.1) between scales, the minimum energy to create such a black hole is again of order the Planck energy. The TeV brane scenario doesn't seem to allow access to what a 4d observer would perceive as strongly coupled gravitational dynamics at lower scales.

In fact, notice the following novelty. A small black hole bound to the Planck brane will not intersect the TeV brane, until its horizon reaches the TeV^{-1} size. Therefore matter moving on the TeV brane may bypass such a black hole, in a close analogy to the black hole flybys discussed in sec. 3.2. (See fig. 2.) In other words, 4d observers made of TeV brane matter have difficulty resolving sub-TeV size black holes. How is this interpreted in four-dimensional language?

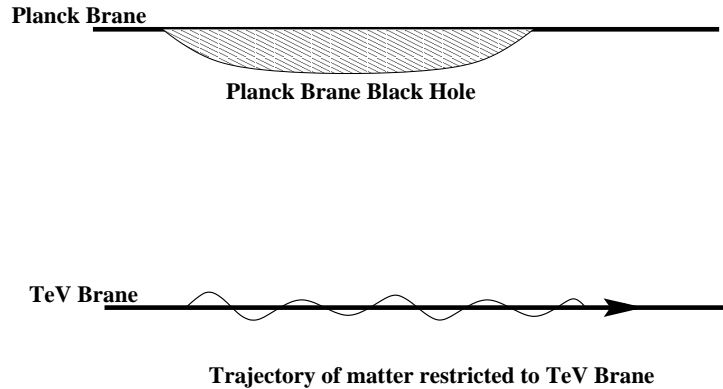


Fig. 2: A particle moving on a probe brane can bypass a small black hole localized on the Planck brane.

⁵ There may be interesting transitory effects – such as stretching and then recoil of the probe brane – that we leave for future investigation.

To really study this question requires a detailed model of the stabilization and the TeV brane matter. However, a plausible answer to this also comes directly from our earlier discussion. Matter passing a black hole by moving on the TeV brane should be interpreted in the 4d perspective as matter smeared out on the TeV scale. Such matter has a small probability of probing a black hole with a radius much less than $1/TeV$.

4.3. Terminated AdS scenario

Another interesting possibility is that AdS is terminated at both ends in z . The outlines of such a picture was suggested in Ref. [3] with a idealized lower brane taken to have a finely-tuned negative tension.

Recent developments in string theory have suggested a concrete means to construct geometries with similar properties. Specifically [6,7] describe geometries that terminate at a definite value of z . These geometries do not arise from negative tension objects, or even singular branes, but rather are rounded off at the maximal z in a smooth geometry that uses the extra dimensions of string theory in a non-trivial way.

Refs. [6,7] do not have a simultaneous microscopic construction of the analog of a Planck brane. However, one can envision building such a model by using Verlinde’s geometric realization[5] of the Planck brane as a piece of a compactification manifold on the ultraviolet end, and then realize the IR brane as in [6,7] or in a variant of these scenarios producing other low-energy dynamics. There may of course be other inequivalent stringy constructions of such doubly-terminated AdS spaces. Constructing detailed models of this kind is an interesting problem for the future.

The models of [6,7] have explicit gauge theory duals. If one constructs a model with a geometric “Planck brane,” one would expect these to be modified at the UV end and depend on the internal structure of the compactification manifold. Nonetheless, these gauge theories should serve as good effective theories at lower scales, in parallel with our earlier discussion.

Such a scenario – or others with a microscopic realization of an IR brane – may have very interesting consequences for the observability of strong gravitational phenomena. Assume that in such a construction there is a gauge theory sector that we may think of as being truly localized in the vicinity of the maximal z which we take to be $z \simeq \rho_T$. This would then realize what was referred to as matter living on the “negative tension brane” of ref. [3]. As explained above, energies at $z \simeq \rho_T$ are redshifted relative to those at the Planck brane, and so if a suitable way is found of stabilizing the separation between

the branes, TeV scale scattering corresponds to a proper energy comparable to M , the five-dimensional Planck scale, if the scattering takes place at $z \simeq \rho_T$. Thus scattering at this scale should begin to make black holes. These should be similar to AdS-Schwarzschild solutions, or to the analogous solutions in the new geometry of [6,7]. (For an explicit formula for the smooth metric in question, see sec. 5.1 of [6].)

In the probe-brane picture, these black holes were expected to fall off the brane and into the horizon at $z = \infty$. Now this is not possible since the geometry terminates at $z \simeq \rho_T$. One expects such a black hole to undergo approximately geodesic motion in this vicinity, and ultimately to evaporate.

Note that one may achieve a clean separation of scales in situations where the AdS radius R is larger than the 5d Planck length M^{-1} . In this situation (which can be achieved by taking large 't Hooft parameter $g^2 M$ – here only M is the dimension of $SU(M)$ – in [6]), there exist black holes larger than the Planck size but smaller than the AdS radius. These would have an approximately (five-dimensional) Schwarzschild description.

In the probe brane picture, the evaporation of 5d black holes was expected to be nearly exclusively into bulk modes, since the black hole becomes well separated from the TeV brane and so will not couple to its excitations. However, in the present picture, the black hole remains in the vicinity of the analog of the IR brane and this suggests that there is no reason for it to decouple from the matter modes in this vicinity. Indeed, in the idealized “negative tension brane” picture, gauge modes on the brane will directly see the black hole metric. Therefore, with this assumption, on general grounds one expects the black hole to Hawking radiate into all available modes, including the visible matter sector modes. As explained in [42], the radiation in the visible sector is generically expected to be important.

This suggests an interesting scenario in which a black hole could be created at an accelerator operating in the vicinity of the TeV scale. Assuming the black hole is sufficiently coupled to the visible modes, these would provide a channel for the Hawking decay and provide an observational window on this process. One would observe such an object by looking for the characteristic approximately thermal spectrum – with increasing temperature – of the Hawking process.

The basic assumptions that could lead to this possibility being realized are 1) that one has a geometry effectively terminated at a maximal z corresponding to the TeV scale, 2) that one has a description of visible-sector matter localized to the vicinity of this maximal z , and 3) that black holes near the maximal z couple to the visible sector. Whether these

assumptions will hold in models based on the ideas of [6,7] remains to be seen, but they plausibly do, and there may also be other models with these properties, for which the creation and visible-sector decay of TeV-scale black holes seems a generic prediction.

5. Conclusions

This paper has investigated the interplay between the four- and five-dimensional descriptions of the physics of warped compactifications. In the simplified example of the RSII scenario, at distances long as compared to the AdS radius R there is a four-dimensional effective description of the dynamics given by observable brane matter coupled gravitationally to a sector described by a conformal field theory. At shorter distances the derivation of this description fails. One expects similar 4d effective descriptions for other warped compactification scenarios.

One element of the correspondence between the 4d and 5d descriptions is supplied by the computation of the 4d stress tensor corresponding to a 5d matter distribution. At the linear level we have given a formula for this stress tensor. We have also investigated an amusing scenario that illustrates the interplay between the 4d and 5d descriptions, that of a particle passing a black hole through the fifth dimension, with a corresponding 4d description in terms of a matter distribution expanding into a shell larger than the black hole.

Finally, we have explored situations in which strong gravitational dynamics may give important modifications to the 4d description. In particular, in scenarios where the hierarchy is addressed by visible matter being effectively localized to a large z in AdS space, one potentially has access to strong gravitational dynamics such as black hole formation at TeV energy scales. In probe brane scenarios this may not lead to observable effects since the resulting black hole seems to rapidly decouple from the visible sector by falling off the brane, but scenarios with AdS terminated at this maximal z show much more promise as such a black hole should stay localized in the vicinity of the maximal z . This leads to the possibility of creation and observable Hawking decay of a black hole in the vicinity of the TeV scale. It would be particularly interesting to find extensions of the work of [5] and [6,7] which give explicit string theory realizations of such terminated AdS scenarios.

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Appendix A. The retarded Green function

In this appendix we describe some properties of the scalar Green function for the RSII geometry. This was given in [11] and takes the form

$$\Delta_{d+1}(x, z; x', z') = \frac{i\pi}{2R^{d-1}}(zz')^{\frac{d}{2}} \int \frac{d^d p}{(2\pi)^d} e^{ip(x-x')} \times \left[\frac{J_{\frac{d}{2}-1}(qR)}{H_{\frac{d}{2}-1}^{(1)}(qR)} H_{\frac{d}{2}}^{(1)}(qz) H_{\frac{d}{2}}^{(1)}(qz') - J_{\frac{d}{2}}(qz_{<}) H_{\frac{d}{2}}^{(1)}(qz_{>}) \right]. \quad (\text{A.1})$$

For the following discussion it is most convenient to use the z coordinate, related to y by (3.1).

The scalar propagator with $d = 4$ and one point on the boundary is given by [11]

$$\Delta_{4+1}(x, z; x', R) = \left(\frac{z}{R}\right)^2 \int \frac{d^4 p}{(2\pi)^4} e^{ip(x-x')} \frac{1}{q} \frac{H_2^{(1)}(qz)}{H_1^{(1)}(qR)}, \quad (\text{A.2})$$

where $q^2 = -p^2$. As in eq. (3.9), let us define a function F ,

$$\Delta_{4+1}(R, x; z', x') = \frac{z'^3}{R} \partial_{z'} \left[\frac{F_{z'}(R; x - x')}{z'^2} \right]. \quad (\text{A.3})$$

Hence, F is given as Fourier transform of Hankel functions,

$$F_{z'}(R; x - x') = -\frac{z'}{R} \int \frac{d^4 p}{(2\pi)^4} e^{ip(x-x')} \frac{1}{q^2} \frac{H_1^{(1)}(qz')}{H_1^{(1)}(qR)}. \quad (\text{A.4})$$

In our conventions (given by (3.6)) the Feynman propagator is

$$\Delta_F(X, X') = -i [\theta(t - t') \Delta^+(X, X') + \theta(t' - t) \Delta^-(X, X')] , \quad (\text{A.5})$$

where Δ^+ is the Wightman function $\langle \phi(X)\phi(X') \rangle$ and Δ^- is its hermitian conjugate. The retarded Green function is defined as

$$\Delta_R(X, X') = -i\theta(t - t') [\Delta^+(X, X') - \Delta^-(X, X')] ; \quad (\text{A.6})$$

this manifestly vanishes for $t < t'$ and can easily be shown to obey (3.6). We therefore find that

$$\Delta_R(X, X') = 2\text{Re}\Delta_F(X, X')\theta(t - t') . \quad (\text{A.7})$$

In order to compute the asymptotic retarded Green function, note that in the long distance approximation ($qR \ll 1$), F reduces to the following form,

$$F_{z'}(R; x - x') \approx -\frac{\pi i z'}{2} \int \frac{d^4 p}{(2\pi)^4} e^{ip(x-x')} \frac{1}{q} H_1^{(1)}(qz') . \quad (\text{A.8})$$

We then perform a Euclidean rotation on the above integral, giving

$$\begin{aligned} F_{z'}(R; x - x') &\approx \frac{\pi i z'}{2} \int \frac{d^4 p}{(2\pi)^4} e^{ip(x-x')} \frac{1}{p} H_1^{(1)}(ipz') \\ &\approx \frac{i}{4\pi^2} \frac{1}{(x - x')^2 + z'^2} . \end{aligned} \quad (\text{A.9})$$

The Feynman prescription is then to replace $(x - x')^2$ with $(x - x')^2 + i\epsilon$. Making this replacement, and taking the real part gives

$$F_{z'}^{Ret}(R; x - x') \approx \frac{1}{2\pi} \delta((x - x')^2 + z'^2) \theta(t - t') , \quad (\text{A.10})$$

which finally yields the retarded scalar propagator,

$$\Delta_{4+1}(R, x; z', x') \approx \frac{1}{\pi R} [z'^2 \delta'((x - x')^2 + z'^2) - \delta((x - x')^2 + z'^2)] \theta(t - t') . \quad (\text{A.11})$$

Appendix B. The infrared limit

As we saw in the text, our calculation of the effective source on the boundary produces an expanding shell in the infrared limit. This is true for scalar, vector, and graviton fields. This is also the result that Horowitz and Itzhaki found in [36], using the boundary conditions appropriate for infinite AdS rather than the brane boundary conditions (3.8). In this appendix we sketch the relation between the calculations. For simplicity we only treat the scalar case although the derivation extends to the other cases.

In our calculation with brane boundary conditions, we solve the bulk equation

$$\square_{d+1}\phi = T , \quad (\text{B.1})$$

with the Neumann condition

$$\partial_n \phi|_{z=\rho} = 0 . \quad (\text{B.2})$$

Here T is the scalar source, in the text given by the falling particle, and ∂_n denotes the normal derivative. The effective boundary source is found by restricting this solution to the boundary and computing its Laplacian:

$$J = \square_d \phi|_{\partial} . \quad (\text{B.3})$$

Another way to get the same solution is to solve (B.1) subject to the Dirichlet boundary condition

$$\phi|_{\partial} = \varphi . \quad (\text{B.4})$$

The solution is given in terms of the Dirichlet Green function as

$$\phi(X) = \int dV' \Delta_{d+1}^D(X, x') T(X') + \oint_{\partial} dn' \partial_{n'} \Delta_{d+1}^D(X, X') \varphi(x') . \quad (\text{B.5})$$

The effective boundary action for φ is computed by evaluating the $d+1$ -dimensional action of this solution, which using the bulk equation of motion becomes

$$S[\varphi] = -\frac{1}{2} \oint_{\partial} dn' \varphi \partial_{n'} \phi . \quad (\text{B.6})$$

The boundary equation of motion for φ then states

$$\partial_n \phi|_{\partial} = 0 . \quad (\text{B.7})$$

Thus a solution of the Dirichlet boundary problem such that the boundary field satisfies the boundary equations of motion corresponds to a solution of the Neumann boundary problem.

In the latter approach the effective boundary source can be read off from the boundary equation of motion. Inserting the second term of (B.5) into (B.7) gives the kinetic operator acting on φ , which becomes \square_d in the long distance limit. Thus in this limit (B.7) states

$$J = \square_d \varphi \propto \partial_z \phi_D|_{\partial} , \quad (\text{B.8})$$

where ϕ_D , the first term in (B.5), is the solution to (B.1) with Dirichlet boundary conditions, $\phi_D|_{\partial} = 0$. In the limit as the cutoff is removed, $\rho \rightarrow 0$, this corresponds to the desired solution in infinite AdS. And aside from a rescaling, $\partial_z \phi_D$ corresponds to the source on the boundary, which if we had been discussing the metric would be the boundary stress tensor of [36].

It is also possible to check the relationship to [36] directly, by acting with \square_d on (3.42), using the eq. (3.6) to eliminate the d -dimensional laplacian in favor of y derivatives, and then using the fact that in the infrared limit (A.1) is the standard bulk propagator plus a y -independent piece which therefore doesn't contribute.

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